Indian Statistical Institute, Bangalore Centre
M.Math. (II Year) : 2010-2011
Semester I : Mid-Semestral Examination
Stochastic Processes

4.10.2010 Time: $2\frac{1}{2}$ hours. Maximum Marks : 80

Note: The paper carries 82 marks. Any score above 80 will be taken as 80.

1. [10 marks] (S, d) is a complete separable metric space, and $(\mathcal{P}(S), \rho)$ denotes the corresponding space of probability measures with the Prohorov metric. For $x, y \in S$ show that $\rho(\delta_x, \delta_y) \leq \min\{1, d(x, y)\}$.

2. [10 marks] Let $S = [0, 1]$ with the usual metric. For $n = 1, 2, \cdots$ let P_n denote the discrete uniform probability distribution on the set $\{\frac{k}{n} : k = 1, 2, \cdots, n\}$. Show that $\{P_n\}$ is weakly convergent and find the limiting probability distribution.

3. [12 marks] (S, d) is a complete separable metric space. Let $X_n, Y_n,\ n = 1, 2, \cdots$ and X be S-valued random variables on a probability space $(\Omega, \mathcal{F}, P)$. Suppose $X_n \Rightarrow X$, and $d(X_n, Y_n) \to 0$ in probability. Show that $Y_n \Rightarrow X$.

4. [8 + 10 marks] (i) (S, d) is a complete separable metric space. If $M \subset B(S)$ is a convergence determining class, show that M is also separating.

(ii) Let (S, d) be a compact metric space. Let $M \subset C_b(S)$ be a separating class. Show that M is also convergence determining.

5. [12 marks] Let $a_n \to a$ in $I\!R$. For $n = 1, 2, \cdots$ let $H_n = \{w \in C[0, 1] : w(0) = 0, w \quad \text{differentiable}, \quad \text{and } |w'(\cdot)| \leq a_n\}$, and let P_n be a probability measure on $C[0, 1]$ such that $P_n(H_n) = 1$. Show that $\{P_n\}$ has a convergent subsequence.

6. [8 marks] Show that the standard Wiener measure is non-atomic.

7. [12 marks] Let $\{B(t) : t \geq 0\}$ be a standard one-dimensional $\{\mathcal{F}_t\}$-adapted Brownian motion. Show that $\{B^2(t) - t : t \geq 0\}$ is an $\{\mathcal{F}_t\}$-adapted continuous martingale.